# Abstract

This paper proposes a data-driven early event identification method based on the measurements of Phasor Measurement Units (PMUs). The central idea is to characterize an event by the lowdimensional subspace spanned by the dominant singular vectors of a data matrix. A dictionary of subspaces that correspond to different events are established off-line, and an event is identified online with the most similar subspace in the dictionary. Both theoretical and numerical analysis demonstrate that the subspace of PMU data is characteristic of system events and is robust to system initial conditions. The size of this dictionary is small and the subspaces can be computed efficiently. The proposed method is evaluated numerically on simulated events in the IEEE 68-bus power systems.

# Introductions

- Fast event identification is beneficial to improve power system security and prevent cascading failures;
- Model-based methods depend on the accuracy of system parameter estimation. and data-driven methods attract more attentions:
- The limitations of existing data driven methods include [1, 2, 3, 4]: Lack of physical interpretations and complicated training models; with a large dictionary size and expensive computations; Most are offline algorithms.

#### Motivations

#### Low-Rank Property of PMU

PMU measurements at m buses accross time T can be approximated by rank-r  $(r \leq m)$  subspaces through singular value decomposition (SVD).

$$M_r = U_r \Sigma_r V_r^{\dagger}, \tag{1}$$

where  $\Sigma_r \in \mathbf{R}^{r \times r}$  contains the r largest singular values,  $U_r \in \mathbf{C}^{m \times r}$ ,  $V_r \in \mathbf{C}^{T \times r}$  span the column and row subspaces respectively.

Event Characterization through Row Subspaces

We motivate the physical interpretation of subspace span( $V_r$ ) through linear system analysis.

$$\mathbf{x}(\mathbf{t} + \mathbf{1}) = A\mathbf{x}(\mathbf{t}) \tag{2}$$

$$\mathbf{y}(\mathbf{t}) = C\mathbf{x}(\mathbf{t}) \tag{3}$$

where  $\mathbf{x}(\mathbf{t}) \in \mathbf{C}^{\mathbf{n}}, \mathbf{y}(\mathbf{t}) \in \mathbf{C}^{\mathbf{m}}$  are deviations of state variables and observations at time t. The measurements  $M = [\mathbf{y}(\mathbf{1}), \mathbf{y}(\mathbf{2}), \cdots, \mathbf{y}(\mathbf{T})]$ can be approximated by

$$M \approx \sum_{k=1}^{r} \mathbf{l}_{k}^{\dagger} \mathbf{x_{0}} \cdot \mathbf{r_{k}} \cdot \beta_{\mathbf{k}}^{\dagger}$$

$$\tag{4}$$

$$\beta_{\mathbf{k}}^{\dagger} = [1, \lambda_k, \lambda_k^2, \cdots, \lambda_k^T]$$
(5)

#### **Remarks of the Property of Subspaces:**

- The row subspace span( $\beta_k$ ) (k=1,..., r) are related to eigenvalues of A, satisfying that the subspace  $\operatorname{span}(\beta_k) = \operatorname{span}(V_r)$  when the approximation of (1) equals (4);
- Row subspaces are independent of the system initial condition  $\mathbf{x}_0$ and thus reduce the dictionary size;

# Fast Event Identification through Subspace Characterization of PMU Data in Power Systems

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# **Outline of Our Method**

The proposed method is outlined in Fig. 1. It is centered on subspacebased event identification, including event detection and location for a practical implementation. The key novelty of our method is that we characterize events by the subspaces and utilize the properties of subspaces to detect, identify and locate events. The specific procedures are described on the right side:



# The Four Components of Our Method

**Offline Dictionary Construction:** We employ the average subspace angle [5] to measure the subspaces similarity as following:

happened.

top k rows.

Figure 1: The flow diagram of the online detection, identification and location method

# Main Ideas of Event Identification through Subspaces



Figure 2:Dictionary construction from historical datasets and real-time data identification through subspace comparison

# Setup of the Simulated Data

The method is tested on the 68-bus power system through power system toolbox (PST). Three types of events described are simulated: load change events (A 0.5 p.u. or 1.5 p.u. step change of active load input at certain bus is used to simulate the abrupt load change by  $ml_sig$  in PST.), line trip events, three-phase short circuit events (cleared after 0.2 second, and the corresponding line is tripped). One second of data after an event starts are selected for analysis, i.e., T = 30, such that the fast dynamics are still maintained after the low-rank approximation. 190 cases including these types at different locations and on different pre-conditions are simulated to construct dictionary and another 380 cases are produced to test our methods. The difference of pre-conditions are measured by the variation of power flow before the event.

$$\theta(V_l, V_k) = \arccos(\sqrt{\|V_k^{\dagger} V_l\|_F^2} / \min\{k, l\}).$$
(6)

where the subspaces spanned by  $V_r, V_l$  with ranks l, k respectively have  $\theta(V_l, V_k)$ . Notice that  $\theta$  equals 0° if two subspaces are the same.

Given the historical datasets, we construct the dictionary of subspaces as shown in Fig. 2. Each subspace is computed efficiently by SVD and the subspace angles between subspaces of the same type are computed and some similar subspaces are greedily removed.

### **Online Detection:**

Let  $\mathbf{y}(\mathbf{t})$  be the measurement at time t, then we project  $\mathbf{y}(\mathbf{t})$  to the column subspace span $(U_r)$ , where  $U_r$  computed by SVD are the basis of column subspace. If the projected error,  $\varepsilon = \|\mathbf{y}_t - \mathbf{U}_r \mathbf{U}_r^{\dagger} \mathbf{y}(t)\|_2$ , exceeds a predetermined threshold, then an event is declared to have

# **Online Event Identification:**

Define the minimum subspace angle  $\theta^*$  with the dictionary of q types of events and  $n_j$  dictionary atoms for type j as

$$\theta^* = \min_{j=1}^{q} \min_{i=1}^{n_j} \theta(V, \bar{V}^{i,j}).$$
(7)

If the minimum is achieved with some dictionary atom  $V^{i^*,j^*}$ , the event is identified to be type  $j^*$ .

### **Online Location:**

The *i*th row of  $U_r \Sigma_r$ , denoted by  $(U_r \Sigma_r)_{i}$ , are utilized to locate events. By sorting buses in a descending order of  $||(U_r \Sigma_r)_{i}||_2$ , we can locate the event approximately within k buses by selecting the corresponding

### The Features of the Event Identification Method

• Characterizing events through subspaces and these subspaces are related to the eigenvalues of state matrix A;

• Only 1 second of post-event data are selected and the subspaces of the data are computed efficiently by SVD;

• As subspaces are robust to initial conditions, the size of dictionary is small;

• Can be implemented in real time.

# **Experiment Results of the Simulated Data**

Table 1: Minimum subspace angles between test cases and dictionary atoms on different pre-conditions

DT Events	Load Change	Line Trip	Short Circuit
Load Change 1	1.03	10.63	18.50
Line Trip 1-30	11.80	0.67	14.32
Short Circuit 5	21.18	11.75	1.97

Table 2:Statistical results of 380 cases with different preconditions

Type of Line Short C Load Ch

- computational methods. 5(6):2947-2956, 2014.
- cluster-based sparse coding.
- Leon M Tolbert. 5(4):1654-1664, 2014.





event	IAR %	ELAR $\%$	ALAR $\%$
Trip	100	85	94(among 3 buses)
Circuit	100	77	90 (among top 3 buses)
hange	100	46	90 (among top 5 buses)

The minimum subspace angles (in bold) between the subspace of events and dictionary in Table 1 demonstrate the type of the events. Table 2 records the identification and location results under three criteria:

Identification Accuracy Rate (IAR): The ratio of the number of accurately identified events to the total number of events;

Approximate Location Accuracy Rate (ALAR): the ratio the number of events with actual locations among the top k buses selected by Step 4 to the total number of events.

Exact Location Accuracy Rate (ELAR): a special case of ALAR when k = 1, i.e., the event location is exact.

All the event types of 380 cases are correctly identified. The accuracy of location rate of serious events like short circuits is higher than that of minor events like load change. The location rates all increase to 90% to 95% if we allow locating the events among 3-5 buses.

# References

[1] Tianzhixi Yin, Shaun S Wulff, John W Pierre, Dongliang Duan, Daniel J Trudnowski, and Matthew Donnelly.

Initial investigation of data mining applications in event classification and location identification using simulated data from minniwecc.

In Proc. North American Power Symposium (NAPS), pages 1–6. IEEE, 2016. [2] Huaiguang Jiang, Jun Jason Zhang, Wenzhong Gao, and Ziping Wu.

Fault detection, identification, and location in smart grid based on data-driven

[3] Yang Song, Wei Wang, Zhifei Zhang, Hairong Qi, and Yilu Liu.

Multiple event detection and recognition for large-scale power systems through

IEEE Trans. Power System, 2017.

[4] Wei Wang, Li He, Penn Markham, Hairong Qi, Yilu Liu, Qing Charles Cao, and

Multiple event detection and recognition through sparse unmixing for high-resolution situational awareness in power grid.

[5] Mahdi Soltanolkotabi, Ehsan Elhamifar, Emmanuel J Candes, et al. Robust subspace clustering.

The Annals of Statistics, 42(2):669-699, 2014.

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